

# A Non-PH Weibull accelerated hazard model

Yasin Al-Tawarah<sup>1</sup> & Gilbert MacKenzie<sup>2</sup>

<sup>1</sup> Royal Medical Services, Amman, Jordan Centre for Medical Statistics , Keele University, Keele, Staffordshire ST5 5BG, UK. E-mail: g.mackenzie @keele.ac.uk

**Abstract:** In this paper we investigate the use of accelerated Weibull hazard model. We compare it with Logistic Accelerated Hazard model. The models are used to analyze survival data from the Northern Ireland lung cancer study and the findings. Maximum likelihood method was used to estimate the parameters of two factors: age and sex.

**Keywords:** Weibull Model, Non-PH model, accelerated hazard

## 1 Introduction

The idea of accelerated hazard survival model mentioned by (Al-tawarah & MacKenzie, 2003, 2004). They discussed a non-PH Logistic Accelerated Hazard (LAH) Model which has a logistic baseline hazard function. It was shown that the marginal fit of the LAH model is better than the Logistic Accelerated Life (LAL) and the PH logistic model. However, the conditional fit was not good enough to support the marginal fit. Thus in this paper we tried to investigate the idea of accelerated hazard for a common model such as the Weibull. We hope the conditional fit will support the marginal fit. Also we compared the Weibull accelerated hazard model with the logistic accelerated hazard model.

## 2 Model Formulation

Assume  $T$  is a non-negative random variable denoted the failure time. The hazard function for weibull distribution takes the form

$$\lambda(t|\lambda, \gamma, x') = \lambda\gamma(\lambda t)^{\gamma-1}\phi \quad (1)$$

where  $\phi = \exp(x'\beta)$  and  $\lambda, \gamma$  are scalars. The baseline hazard function is

$$\lambda_0(t|\lambda, \gamma) = \lambda\gamma(\lambda t)^{\gamma-1} \quad (2)$$

and the survival function defined as

$$S(t|\lambda, \gamma, x') = \exp[-(\lambda t)^\gamma \phi] \quad (3)$$

Thus one can accelerate (2) and obtain the Weibull accelerated hazard function as

$$\lambda(t|\lambda, \gamma, x') = \lambda\gamma(\lambda t\phi)^{\gamma-1} \quad (4)$$

which on integrating yields to the corresponding survival function as

$$S(t|\lambda, \gamma, x') = \exp[-(\lambda t)^\gamma \phi^{\gamma-1}] \quad (5)$$

whence the resulting density function is

$$f(t|\lambda, \gamma, x') = \lambda\gamma(\lambda t\phi)^{\gamma-1} \exp[-(\lambda t)^\gamma \phi^{\gamma-1}] \quad (6)$$

We compare this model with the logistic accelerated hazard model defined by (Al-tawarah & Mackenzie, 2003, 2004) which takes the form

$$\lambda(t|\alpha, \lambda, x') = \frac{\lambda \exp(t\alpha\phi)}{1 + \exp(t\alpha\phi)} \quad (7)$$

### 3 Model Fitting

First we fit the Weibull accelerated model using survival data from the Northern Ireland lung cancer study (Wilkinson, 1995). This was a prospective study designed to measure the survival of incident cases of lung cancer. During that year 900 incident cases were diagnosed and followed up. We obtained the maximum likelihood estimate for age and sex. We also obtained the marginal and conditional fits for both factors. We compare the marginal and conditional fits obtained for Weibull regression model with the marginal fit of the Kaplan & Meier estimator. Moreover, we compare this regression model with the accelerated hazard logistic model for the same factors.

### 4 Results

Table ( ??) shows the maximum likelihood estimate of the regression parameters for both models. We note here that the singe of the parameter estimate is opposite of the singe estimate in Cox regression fit (not shown). So the interpretation is that the positive singe of the parameter increases the survival or decreases the hazard. The marginal fit of the LAH hazard model shows a better fit compared with the Weibull model when we compare both of them with KM estimator (Figures 1, 3). The conditional fit of both models are almost closed.

TABLE 1. Comparison of Maximum Likelihood Estimate: Age and Sex.

Factors	Parameters	Weibull		LAH	
		Estimate (Weibull)	$\hat{SE}$	Estimate (LAH)	$\hat{SE}$
Age	$\hat{\lambda}$	0.031	0.013	0.240	0.055
	$\hat{\gamma}$	0.863	0.026		
	$\hat{\beta}_1$	-0.128	0.042	-0.020	0.112
	$\hat{\alpha}$			-0.346	0.241
Sex	$\hat{\lambda}$	0.123	0.009	0.250	0.054
	$\hat{\gamma}$	0.853	0.027		
	$\hat{\beta}_1$	-0.224	0.577	-0.157	0.218
	$\hat{\alpha}$			-0.096	0.018
Age	$\hat{\lambda}$	0.030	0.013	0.319	0.055
	$\hat{\gamma}$	0.864	0.026		
	$\hat{\beta}_1$	-0.128	0.042	-0.020	0.110
	$\hat{\alpha}$				
Sex	$\hat{\beta}_2$	-0.129	0.603	-0.141	0.218
	$\hat{\alpha}$			-0.368	0.257

## 5 Summary

The idea of an accelerated hazard model is new. We see here that common Weibull model does not show better fit comparing with the LAH model. So we expect that the LAH model may have some applicable properties if applied into different data set.

## References

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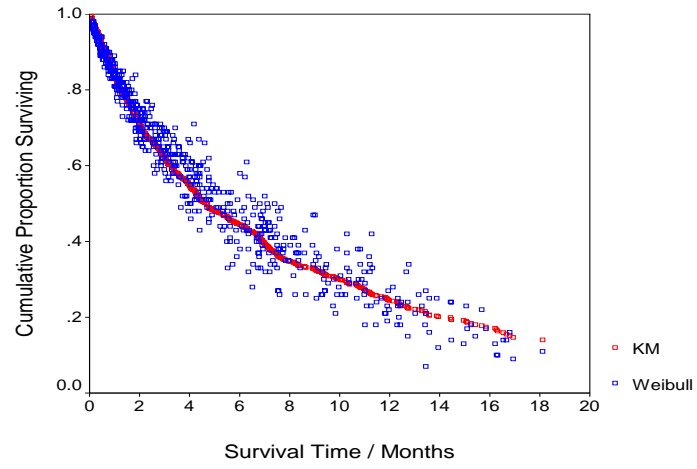


FIGURE 1. Predicted Weibull v KM .

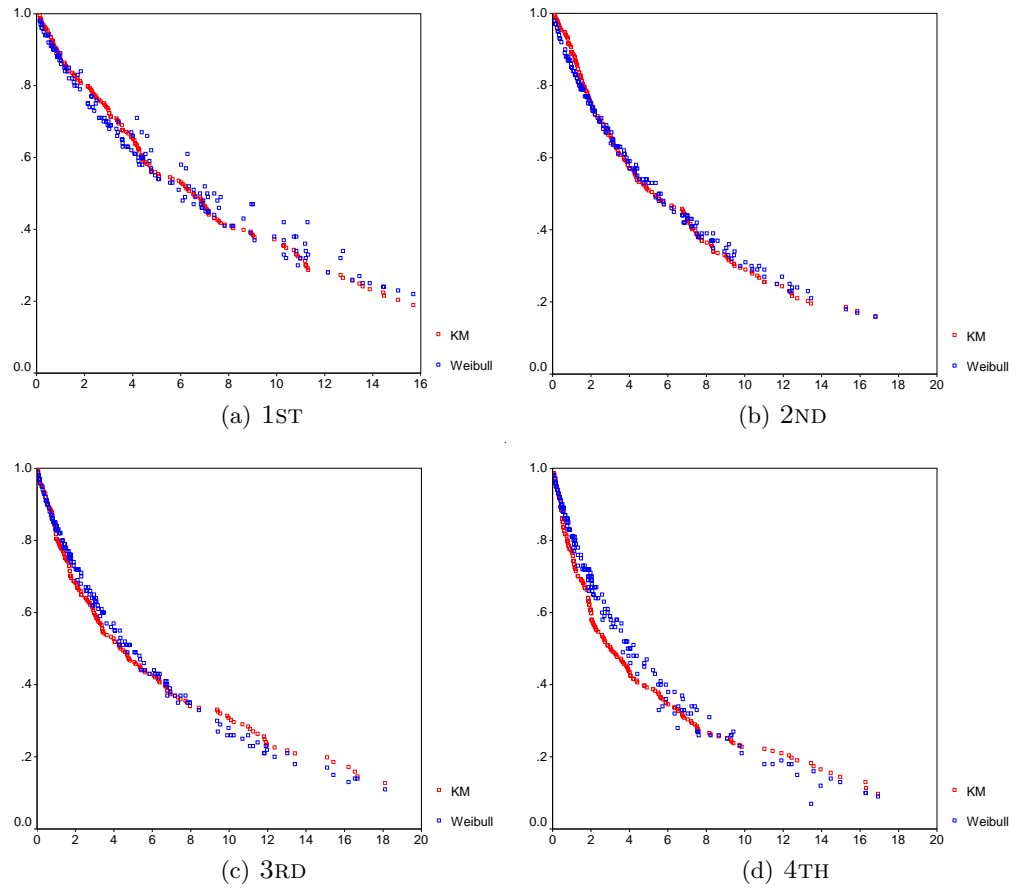


FIGURE 2. Predicted Quartiles Weibull v KM .

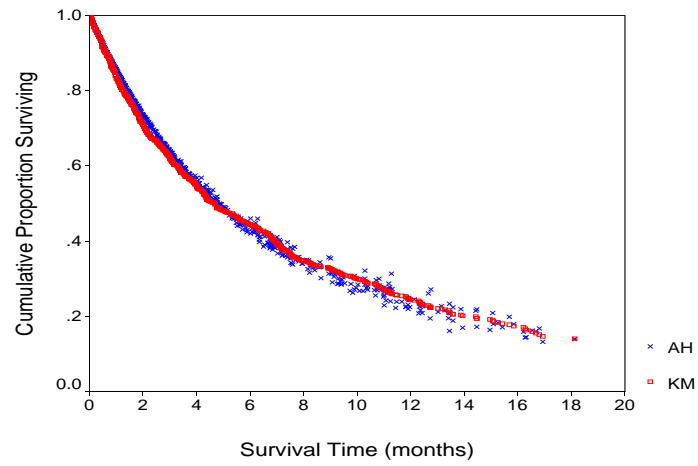


FIGURE 3. Predicted LAH v KM .

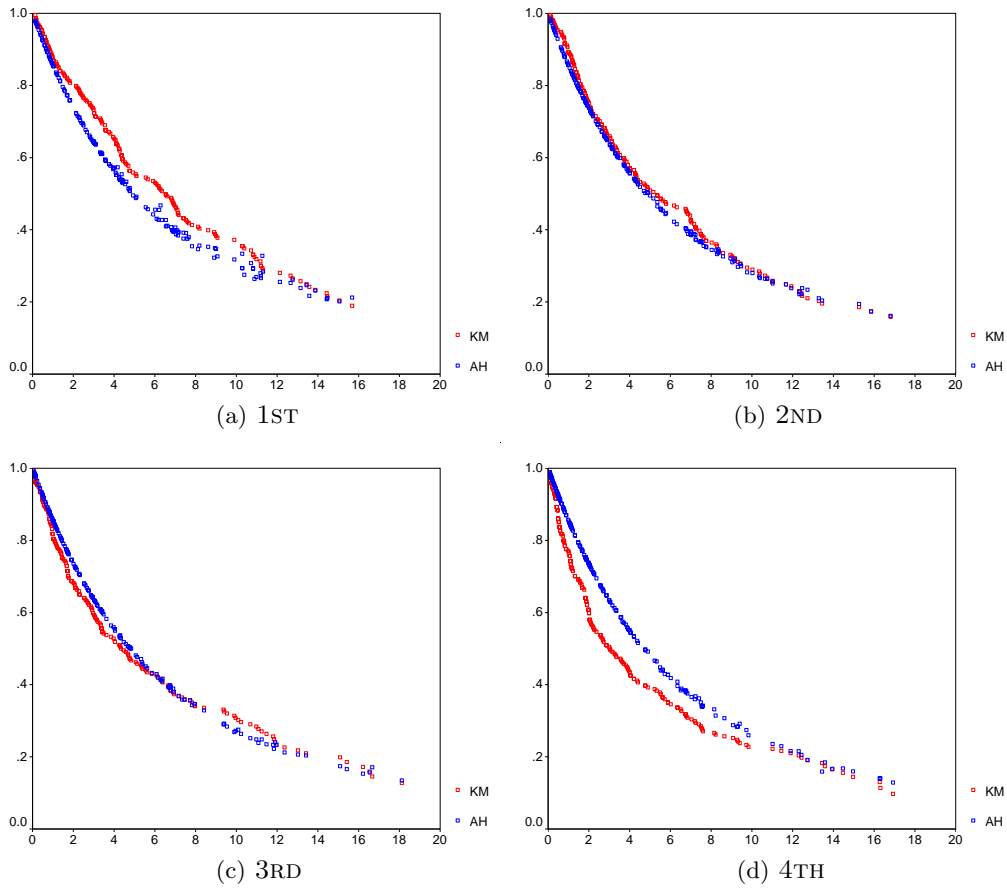


FIGURE 4. Predicted Quartiles LAH v KM .